AD-A095 559 AERONAUTICAL RESEARCH ASSOCIATES OF PRINCETON INC NJ F/G 12/1
STUDIES IN NONLINEAR MOTION.(U)
JAN 81 H SEGUR DAAG29-78-C-0003
UNCLASSIFIED ARAP-434 ARO-15338.8-M NL

(18) (19) ARO 15338.8-M

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A095559	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Studies in Nonlinear Motion. (12)	5. TYPE OF REPORT & PERIOD COVERED Final Report 1/1/78 - 12/31/80
	6. PERFORMING ORG. REPORT NUMBER A.R.A.P. Report No. 434
7. AUTHOR(s) (10) Harvey Segur (15)	8. CONTRACT OR GRANT NUMBER(s) DAAG29-78-C-0003
9. PERFORMING ORGANIZATION NAME AND ADDRESS Aeronautical Research Associates of Princeton, Inc. 50 Washington Road, P.O. Box 2229 Princeton, New Jersey 08540 (16)	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1L161102BH57-05 Math
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709 (11)	12. REPORT DATE January 1981 (17) 05
	13. NUMBER OF PAGES 5
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) LEVEL II (12) 5	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

(9) Final rept. 1 Jan 78 - 31 Dec 80,

DTIC ELECTE FEB 24 1981 D C

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

(14) ARAP-434

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ordinary differential equations, Painlevé, Bäcklund transformations, nonlinear evolution equations, Inverse Scattering Transform (IST)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The most important result of this research has been the development and exploitation of the connection between nonlinear evolution equations solvable by some Inverse Scattering Transform (IST) and ordinary differential equations of Painlevé-type. This connection provides a test (or series of tests) which a given evolution equation must pass if it is solvable by IST. Being a necessary condition, it complements the various sufficient conditions that already were known. Independently, the connection also can be used to obtain global informa-

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DATE 81 2 18 179

AD A095559

DDC FILE COPY

tion about the classical Painlevé transcendents. This information becomes increasingly important as these equations are discovered more and more frequently in models of physical problems.

STUDIES IN NONLINEAR MOTION

Final Report

Contract No. DAAG29-78-C-0003

Harvey Segur

As outlined in the original proposal, the objective of the research supported by this contract was to study certain problems relating to evolution equations that are completely integrable. The problems identified in that proposal were:

(a) Defining the class of integrable evolution equations;
(b) Bäcklund transformations;
(c) Asymptotic behavior of solutions; and
(d) Optimal numerical procedures.

The research resulted in eight publications in the open literature, which are listed in the References. (The subsequent discussion will refer to these papers by their numbers in this list.) These papers are related to the original problems posed in the following way.

(a) Defining the class of integrable evolution equations.

This aspect of the research program has been extremely successful. We now know that there is a deep connection between partial (or ordinary) differential equations that are completely integrable and ordinary differential equations of Painlevé-type. Three years ago this connection had only recently been discovered, and much of the work done under this contract has been aimed at discovering exactly what the connection is, and to what extent it can be exploited. (At least part of References 2-8 focus on this connection in some way or other.) By now we have a strong practical knowledge of what the connection is, and we have shown that it is a very effective tool to identify both partial and ordinary differential equations that are completely integrable (see References 2, 3, 5, 7, and 8). It may also be used to obtain global information about the classical Painlevé transcendents (References 3, 4, 6, and 8). We do not yet understand the connection well enough to prove rigorously that it always exists. In fact, for ordinary differential equations, we cannot even say why one should expect such a connection. Work in this direction is continuing.

(b) Bäcklund transformations.

One of the original objectives of the research proposed was simply to define a Bäcklund transformation precisely. This was accomplished and is the basis of Section 3.1 in Reference 8 on Bäcklund transformations. In addition, the very restricted range of the Miura transformation (perhaps the most famous Bäcklund transformation) was pointed out in Reference 1.

(c) Asymptotic behavior of solutions.

The complete asymptotic behavior of three integrable partial differential equations (KdV, mKdV, sine-Gordon) was given in Reference 4. Earlier analyses of these problems that had been published were either incomplete (not necessarily a serious objection), incorrect (more serious), or both.

The asymptotic behavior of the Toda lattice was never studied seriously. That project became less important when we realized that the Toda lattice, or any integrable lattice, has no interesting thermodynamic properties by itself. For example, regardless of the motion of its masses, the thermodynamic temperature in a single Toda lattice is always zero.

(d) Optimal numerical procedures.

Because of changes in personnel at A.R.A.P., no work on optimal numerical procedures ever was initiated. Work on this problem has been initiated by a graduate student of Mark Ablowitz at Clarkson College, and I have helped to direct his study.

In the opinion of the Principal Investigator, the most important result of this research has been the development and exploitation of the connection between nonlinear evolution equations solvable by some Inverse Scattering Transform (IST) and ordinary differential equations of Painlevé-type. This connection provides a test (or series of tests) which a given evolution equation must pass if it is solvable by IST. Being a necessary condition, it complements the various sufficient conditions that already were known. Independently, the connection also can be used to obtain global information about the classical Painlevé transcendents. This information becomes increasingly important as these equations are discovered more and more frequently in models of physical problems.

References

1. Ablowitz, M. J., Kruskal, M., and Segur, H., "A Note on Miura's Transformation," J. Math. Phys. 20, 999 (1979).

2. Ablowitz, M. J., Ramani, A., and Segur, H., "Nonlinear Evolution Equations and Ordinary Differential Equations of Painlevé Type," Lettere Nuovo Cimento 23, 333 (1978).

3. Segur, H., "Ordinary Differential Equations of Painlevé-Type and the Inverse Scattering Transform," in Mathematical Methods and Applications of Scattering Theory: Proceedings of a Conference Held at Catholic University, Washington, D.C., May 21-25, 1979, J. A. DeSanto, A. W. Saenz, and W. W. Zachary (eds), Springer-Verlag (New York), 1980, pp. 255-259.

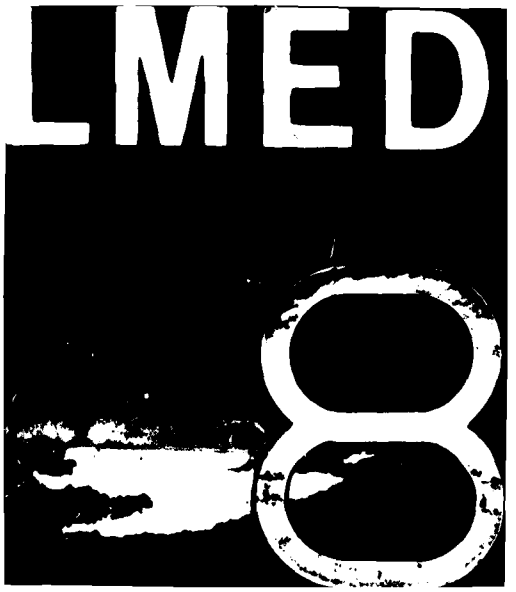
4. Segur, H., and Ablowitz, M. J., "Asymptotic Solutions of Nonlinear Equations and a Painlevé Transcendent," presented at the Joint US-USSR Symposium on Soliton Theory, Kiev, USSR, September, 1979.

5. Ablowitz, M. J., Ramani, A., and Segur, H., "A Connection between Nonlinear Evolution Equations and Ordinary Differential Equations of P-Type, I," J. Math. Phys. 21, 715 (1980).

6. Ablowitz, M. J., Ramani, A., and Segur, H., "A Connection between Nonlinear Evolution Equations and Ordinary Differential Equations of P-Type, II," J. Math. Phys. 21, 1006 (1980).

7. Segur, H., "Solitons and the Inverse Scattering Transform," Lectures presented at the International School of Physics, "Enrico Fermi," Varenna, Italy, July 7-19, 1980.

8. Ablowitz, M. J., and Segur, H., Solitons and the Inverse Scattering Transform, to be published by SIAM press, 1981.


LMED
8